Novel evaluation of bender response in sand specimens using a laser vibrometer

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ABSTRACT
Bender elements (BE) are used to measure the shear-wave velocity of soils in many geotechnical laboratories worldwide. Despite its popularity, the method remains without a standard procedure, mainly because of the difficulties in controlling the actual behavior of the BE inside the soil specimen. Previous studies have monitored and/or simulated the behavior of BE, finding evidence that the actual transmitter movement is not equal in shape and frequency content of the electrical signal used as excitation. However, this is not supported by any direct measurements of BE transmitter vibrations. In this study, a transparent soil is used in conjunction with a laser vibrometer to obtain actual BE transmitter movements in the transparent soil. Measurements of transmitter BE are obtained in air and in transparent soil under five different stresses. Results show that the BE response in air with a sine pulse excitation resembles the response of a single-degree-of-freedom (SFOD) system. However, the BE response in soil is significantly different with the same excitation. Moreover, the BE response in the transparent soil shows an unexpected variation in amplitude with increase in stress; whereas, there is an increase in frequency of vibration that is consistent with the increase in the medium stiffness.

RÉSUMÉ
Les «elements bender» (BE) sont utilisés pour mesurer la vitesse de propagation des ondes de cisaillement à faible déformation dans les sols dans de nombreux laboratoires à travers le monde. Malgré sa popularité, cette méthode ne possède pas de procédure normalisée, principalement à cause des difficultés rencontrées dans le contrôle du comportement des BE dans l'échantillon. Des études expérimentales et analytiques précédentes ont mesuré et/ou simulé le comportement des BE, prouvant que la transmission de mouvement n'est pas équivalente au signal électrique utilisé en excitation en termes de forme et de fréquence. Cependant, cette hypothèse n'est pas appuyée par des mesures directes du comportement des transmissions des BE dans un échantillon de sol. Dans la présente étude, un sol granuleux transparent est utilisé avec un laser vibromètre pour obtenir les transmissions de mouvement des BE dans l'échantillon de sol transparent. Les mesures des transmissions des BE sont obtenues dans l'air et dans le sol transparent sous cinq pressions verticales différentes. Les résultats démontrent que la réponse des BE dans l'air, à une pulsation sinusoïdale, ressemble à la réponse d'un système à un seul degré de liberté (SFOD). Cependant, la réponse des BE dans le sol avec la même excitation est significativement différente. De plus, la réponse des BE dans le sol transparent démontre une variation en amplitude et une augmentation de la fréquence de vibration avec augmentation de la pression de confinement.

1 INTRODUCTION

Bender elements (BE) are commonly used to measure shear wave velocity of soils (V_s) at low shear strain levels (γ). These piezoceramic transducers have been used with numerous geotechnical equipment since they were first proposed (Shirley & Hampton, 1978). A voltage signal is applied to the transmitter, which generates a small perturbation (mechanical energy) in the specimen that travels from one side of the specimen to the other. On the other side, a receiver converts the mechanical energy into output voltage. The peak-to-peak distance between the transducers and the travel time of the perturbation from transmitter to receiver are used to estimate V_s of the material (Dyvik & Madshus, 1985). The BE methodology still remains without a standard procedure because of the difficulties in controlling the actual behavior of BE inside the specimen.

BE have been typically combined with geotechnical equipment used for stress/strain measurements such as triaxial cells, oedometers, and resonant-column device. Wave propagation phenomenon in BE has been studied experimentally and theoretically. The reliability of BE results is influenced by several factors such as cross talk and directivity; boundary effects; sample geometry and size (Arroyo et al., 2006; Arulnathan et al., 1998; Lee & Santamarina, 2005). Viggiani & Atkinson (1995) used the first inversion of the received signal to determine travel time; which gave more consistent results than the results from cross correlation or phase analysis of cross-power spectrum (Santamarina & Fam, 1997). Moreover, Jovicic et al., (1996) proposed the use of square waves as the excitation in combination with the first inversion of the received signals to get more accurate results. Brignoli et al., (1996) found that first inversion is not always accurate and the travel time should be identified using the shape of the received signals which may include near field effects.

Other methods in time and or frequency domain have been proposed to interpret BE tests accurately (Greening & Nash, 2004). Camacho-Tauta et al., (2008) used sine wave pulses at different frequencies to estimate the first arrival; and then analyzed the results from sinusoidal sweep excitations in different frequency ranges.

The actual behavior of BEs in a soil specimen is still not well understood. Limited experimental programs have been conducted to evaluate the BE behavior inside the soil specimen. This study is part of a novel experimental program that focuses on the evaluation of the actual BE behavior inside the soil specimen. A transparent soil specimen with mechanical properties similar to those of granular soils with angular particles is used in conjunction with a state of the art laser vibrometer (Ezzein & Bathurst, 2011). The BE responses in air are compared to those in the transparent soil. Moreover, measurements are obtained in transparent soil under five different horizontal stresses.

2 BACKGROUND

2.1 Electrical input and actual movement of the BE

In a BE test, it is typically assumed that the frequency contents of the electrical input signal and the actual response of the transmitter BE are the same. However, measurements performed by different authors indicate otherwise (Lee & Santamarina, 2005; Pallara et al., 2008). Pallara et al., (2008) used a laser vibrometer to show that there is a time delay between the input signal used for excitation and the actual movement of the bender transmitter; and that the actual displacement is non-symmetric unlike the input signal. Rio (2006) used a laser vibrometer and a synthetic rubber specimen to measure the BE response under sinusoidal-pulse and sine-sweep excitations in air and embedded conditions. The laser measurements were obtained through a small hole in the rubber specimen. He showed that the response of the BE transmitter under free conditions is governed by its natural frequency rather than excitation frequency. Under embedded conditions, the amplitude of vibration decreased while damping ratio and natural frequency of transmitter increased. However, the effect of the particle behavior of a typical soil on the BE response is still not known. This study presents the actual BE behavior in a transparent soil which exhibits mechanical properties similar to angular sands (Ezzein & Bathurst, 2011).

2.2 Transparent soil

The transparent soil used in this study was developed by Ezzein & Bathurst (2011). It is made up of hard fused quartz and two mineral oils. Fused quartz is a noncrystalline glass (SiO_2). It is manufactured by melting natural quartz crystals from quartzite at around 2000 °C followed by cooling (Weast et al., 1982). The two mineral oils used are Krystol-40 and Puretol-7 manufactured by Petro-Canada Lubricants. The underlying principle in mixing the oils is to match the refractive index of the fused quartz (32% Kryston-40 and 68% Puretol-7). In this study, coarse grained transparent soil was used (D_{50} = 1.688 mm) to improve the resolution of the laser measurements.

2.3 Vibration of BE system

The vibration behavior of a BE (resonance frequencies and mode shapes) is assumed to be similar to the response of a cantilever beam with fixed-free boundary conditions. The resonant frequency of the *nth* mode of a cantilever beam can be used to estimate the resonant frequency of the BE by using the equation (Clough & Penzien, 2003)

$$f_n = \frac{k_{Ln}^2}{2\pi(\alpha L_b)^2}\sqrt{\frac{E_b I_b}{\rho_b A_b}} \qquad [1]$$

where k_L is a characteristic number which depends on n and the boundary conditions; L_b, I_b and ρ_b are the length, area moment of inertia ($I_b = bh^3/12$), and mass density of BE respectively; b, h, and A_b are the width, thickness, and cross-sectional area ($A_b = bh$) of the BE respectively; E_b is the Young's modulus of the piezoceramic element; α is the effective length factor where $\alpha = 1$ for perfectly fixed conditions and $\alpha > 1$ for flexible conditions. On the other hand, the resonant frequency of the first mode of vibration of a BE under embedded conditions can be estimated using the equation (Lee & Santamarina, 2005)

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1.875^4\dfrac{E_b I_b}{(\alpha L_b)^3} + 2\eta V_s^2 \rho_s (1+\nu) L_b}{\rho_b A_b \alpha L_b + (\rho_s b^2 L_b)\beta}} \qquad [2]$$

where ρ_s and υ are the mass density and Poisson's ratio of the soil respectively; β is the experimental factor related with the volume of soil affecting the vibration of BE; and $\eta \approx 2$ is the mean displacement influence factor at the soil-BE interface.

3 EXPERIMENTAL SETUP

Two experimental setups were used in this study. The BE used in this study was fixed to a steel top cap (TC) of a resonant column setup (Cascante et al., 2005). In the first setup, the BE vibrations in air are measured as shown in Figure 1. The top platen is attached to an *x-y* stage that allows precise location of the BE with a resolution of 1 micrometer in the x and y directions. The laser sensor head (LSH) and the BE system are placed on an isolating table to avoid noise from ground vibrations to affect the measurements.

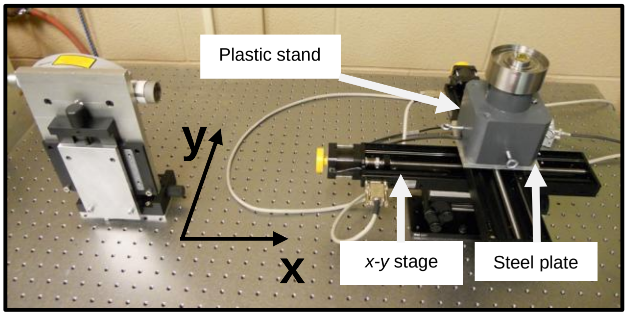

Figure 1 - Photograph of setup for BE measurements in air – platen facing up

The second experimental setup was used to measure the vibrations of the transmitter BE embedded in the transparent soil (Figure 2). The BE vibration measurements in soil are obtained in stages to evaluate the effect of different components. First, the BE vibrations are measured in air through the wall of the Plexiglas container; then with the BE submerged in oil only, and finally with the BE embedded in the transparent soil. The electronic equipment used is also shown in Figure 2. A function generator (FG) (model HP33120A) is used to generate the low-voltage input signal (5 volts peak-to peak); which is amplified by a power amplifier or piezo-driver (PD) by a factor of 20. The input and output signals are monitored with a digital oscilloscope (DO) (HP-54645A). The laser sensor head (LSH) is connected to the vibration controller (OFV-5000) which acts as a decoder of the laser measurements. The laser system stores the input and output signals in the computer. The system is capable of measuring displacements with frequency range of up to 24 MHz (Polytec, 2013).

4 EXPERIMENTAL METHODOLOGY

The laser vibrometer requires a special reflecting paper attached on the target to measure the vibrations. LSH is fixed on to isolated table. The distance *X* in Figure 2 between the LSH and BE is kept fixed for all the experiments.

4.1 BE measurements in air

The TC is placed facing up on the plastic stand. BE vibration measurements are taken at three points. First, the laser beam is placed at the center of BE vertically and horizontally. For the other two points, the steel plate is moved horizontally from the center of BE by about 0.5 mm in *y* direction (Figure 1). The *x-y* stage is used to move the steel plate from the center to the right of BE and to the left. At each point, three different input excitation pulses are applied to the BE; 50 Hz square, 11 kHz square, and 11 kHz sine

4.2 BE measurements in transparent soil sample

BE measurements in the transparent soil are obtained in steps. All measurements are obtained at the center of BE with an input excitation sine pulse of 11 kHz. The TC is rotated such that the laser beam becomes perpendicular to the BE face.

First, the TC is placed facing down on a plexiglass tube of about 5 cm diameter and 3 cm height. The tube is placed on the steel plate. About 0.15 cm of circumference of the tube is cut off to prevent scattering of the laser beam caused by the face of the tube. This position of the TC resembles the way in which the TC is placed in a typical resonant column setup (Cascante et al., 2005). In the next step, the same plexiglass tube is placed in a plexiglass box of dimensions 8 cm x 7 cm x 12 cm. The box is tied to the steel plate using elastic bands. Measurements are again obtained for BE vibrations in air but with the laser beam now going through the plexiglass box. With TC and tube placed in their position, the mineral oil mixture (hereafter referred to as ’oil’) is poured in the plexiglass box until the BE is covered by the oil. Measurements are taken to observe the effect of the oil on BE vibrations. Then, the plexiglass tube and TC are removed to add the fuzed quartz in the plexiglass box to form the transparent soil sample. TC is placed on to the transparent soil sample in the plexiglass box (Figure 2) and the measurements are taken. A bubble level is placed and rotated on the TC to ensure that the TC is not put unevenly on the transparent soil.

4.3 Measurements with increasing vertical stress

BE vibrations are obtained for a total of five vertical stresses. The stress is applied in increments of 2.5 kg loads on the top platen through a plexi-glass tube. The stress on the transparent soil is estimated using Boussinesq (1885) theory for a uniformly loaded circular area. The first level of stress is applied using the weight of the top platen and the plexi-glass tube, and the subsequent levels of stress are incrementally applied through four 2.5 kg loads. Figure 4 shows a schematic of the setup with the final level of stress applied.

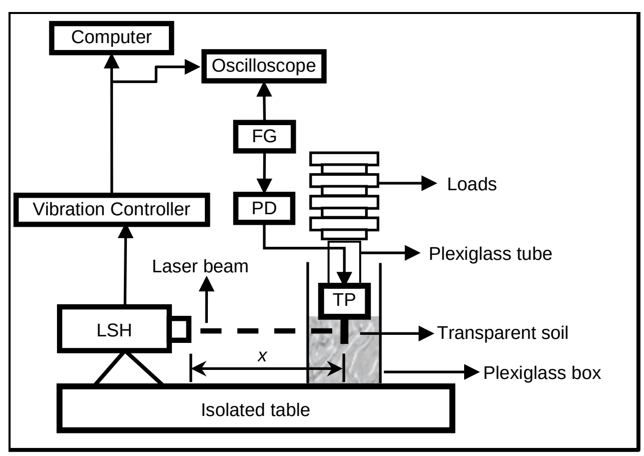

Figure 2 - Schematic of setup BE measurements in soil with confinement

5 RESULTS AND DISCUSSION

All measurements presented are obtained at the center of BE unless noted otherwise. The time signals presented in this section are normalized. The vertical scale for the respective normalization is shown in the figures. Moreover, the input signals shown in the figures of time signals are only shown for a comparison of arrival times. The amplitudes of these input signals are not accurate. Fast Fourier Transform (FFT) magnitudes presented are also normalized. However, the FFT magnitudes are not absolute, but are presented to show the ratio of one magnitude to the other.

5.1 BE measurements in air

BE vibration responses to different input excitation pulses are measured to determine the effect of using different pulse types as the input signal on the BE. Figures 3 and 4 show the time signals and FFT magnitude, respectively, for the BE response to the three input excitations. Figure 3 shows that both the square pulses induce high frequencies initially after which a damped free vibration response is observed. The 11 kHz sine pulse also shows a damped free vibration response with barely any high frequencies.

FFT magnitudes in Figure 4 are normalized to FFT magnitude corresponding to 11 kHz sine pulse. Figure 4 shows that the peak FFT magnitude for all the three input excitations corresponds to a frequency of about 11 kHz. Hence, the resonance frequency of the first mode of vibration (f_1) of BE is about 11 kHz. The damped free vibration responses shown in Figure 3 have a frequency of 11 kHz. The magnitude of the response to the 11 kHz square pulse at f_1 is about 2.7 times larger than that of 50 Hz square pulse. The magnitude of the response to the 11 kHz sine pulse at f_1 is around 2 times higher than that of 50 Hz square. The difference in FFT magnitudes to the square pulses is because the central frequency (f_c) of the 50 Hz square pulse is significantly different from f_1 of BE response in air. The difference in FFT magnitudes corresponding to 11 kHz sine and square pulses is explained by considering the power spectra of different pulses (Tallavo et al., 2009). The square pulse has the maximum power spectrum magnitude than triangle, sine and sawtooth pulses. Hence, the BE response corresponding to the 11 kHz square pulse has the highest amplitude.

The FFT magnitudes corresponding to both the square pulses have peaks at frequencies higher than f_1 (Figure 4). These might correspond to resonance frequencies of higher modes of BE free vibration. Equation [1] is used to estimate f_2 of BE to determine if the second peak in Figure 4 corresponds to the second mode of BE vibration. Properties of BE used for estimating f_2 are: E_b=6.3x10^{10} N/m², ρ_b=7700 kg/m^3, L_b=4.5 mm, b=10 mm and h=0.6 mm (Ismail & Hourani, 2003). The value of α = 1.12 was estimated using f_1 = 11 kHz and k_L = 1.875 for the first mode. The estimated value of f_2 was about 68 kHz. Hence, the peaks at higher frequencies in Figure 4 do not correspond to higher modes.

The above comparison of response to different input excitations shows that the square wave pulse excitations induce high frequencies in BE response. These high frequencies do not correspond to the BE response as shown by the estimation of f_2. Hence, the 11 kHz sine pulse excitation is used for the remainder of the results.

Measurements are taken at three points on the BE. Time signals and FFT magnitudes of the BE response to 11 kHz sine pulse at the three points; center, left, and right are presented in Figure 5 and 6, respectively. Magnitude at f_1 of the point in center and the point on the left of center is about 1.7 times and 2.3 times higher respectively than that of point on right. Additionally, the time signals in Figure 5 show a difference in phase between the three points. The unwrapped phase of the three signals is obtained to estimate the phase difference ($\Delta\varphi$) between these signals at f_1. $\Delta\varphi$ between the point at the center and on the right was minimum. However, the $\Delta\varphi$ between the point on the left and center (and right) was about 0.946 radians. Hence, BE vibrates as a flexible medium as opposed to what is assumed in BE tests (Lee & Santamarina, 2005). A subsequent study will be performed on experimental modal analysis of BE in air and in transparent soil to characterize these differences

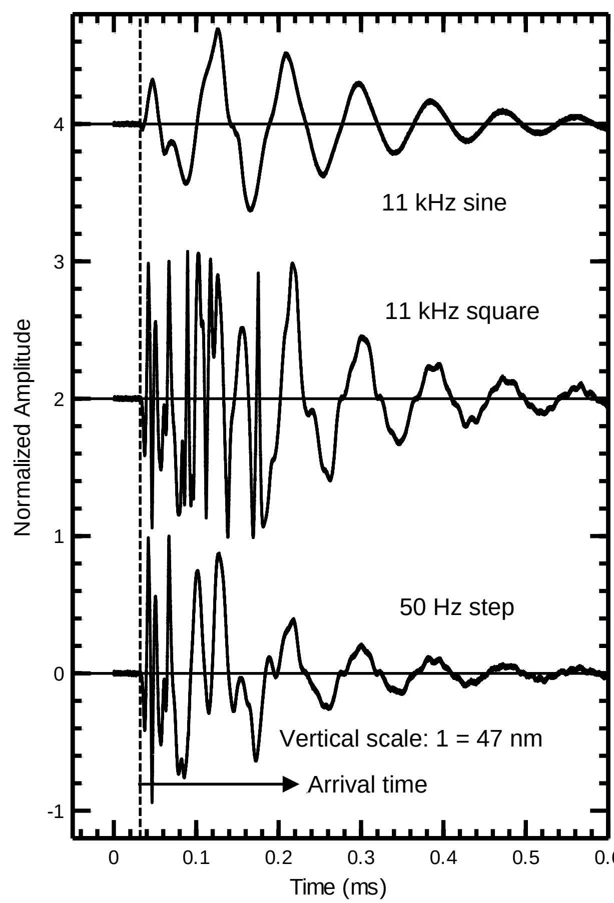

Figure 3 - BE response to different excitation pulses

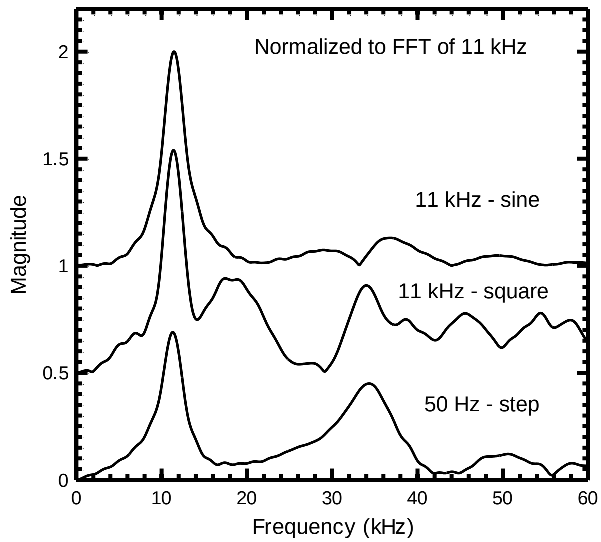

Figure 4 - FFT magnitude of BE response to different input signals

5.2 BE response in transparent soil

The transparent soil sample is placed in a plexi-glass box. Several obstructions are present between the laser beam and BE in transparent soil; plexi-glass box, mineral oil, and fused quartz. Measurements for BE vibrations are obtained in stages after adding each obstruction. Figure 7 and 8 show the time signals and FFT magnitude plots of BE vibrations in three different media. The responses in 'air' and in 'air with plexiglass' are very similar. Presence of oil significantly dampens the 11 kHz BE vibration. Magnitude at resonance frequency of BE vibration in oil is about 40 % of that in air. Magnitudes for higher frequencies have also been reduced with the presence of oil. Moreover, phase difference between BE vibration in oil and in air at resonance frequency is about 1.63 radians.

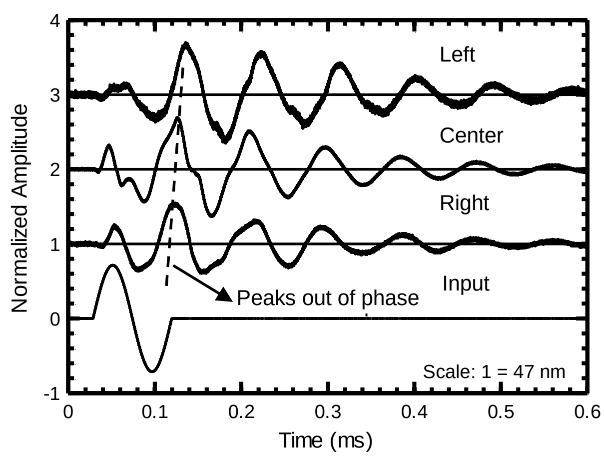

Figure 5 - Comparison of different points on BE for 11 kHz sine pulse excitation

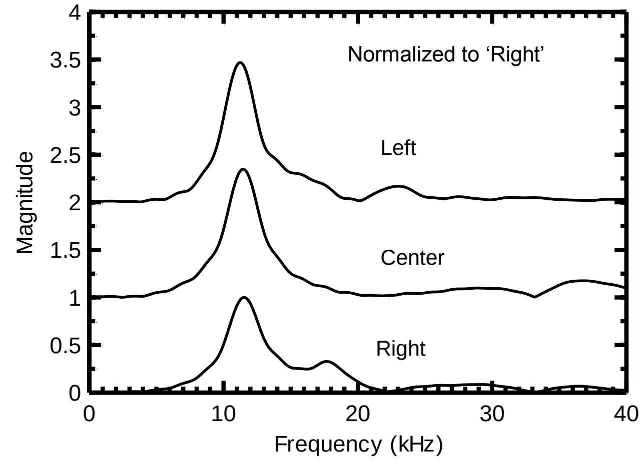

Figure 6 - Frequency spectra of response at different points

Figure 8 shows that the resonance frequency of BE vibration in oil is about 8.4 kHz which is less than the resonance frequency in air. The mineral oil used in transparent soil is ten times more viscous than water (Ezzein & Bathurst, 2011). Moreover, the viscous fluid represents an added mass on the BE. The combine effect of viscosity and added mass is reducing the amplitude and natural frequency of BE vibration. This effect will be further analyzed in a future study as part of this experimental program.

The BE response in transparent soil is compared with that in air in Figure 9 and 10. Presence of fused quartz combined the mineral oil has increased the amplitude at resonance frequency of BE vibration. The amplitude of BE vibration in soil is about 50 % of the amplitude in air; 10 % higher than amplitude in oil. The value of f_1 = 11.01 kHz in soil is higher compared to f_1 = 8.4 kHz in oil; an increase of 2.6 kHz. Figure 10 also shows that the region around the peak for FFT magnitude of soil is broader than those of oil and air. BE immersed in a soil specimen is expected to vibrate at a higher frequency with lower amplitude compared to BE vibration in air. The presence of quartz in combination with the mineral oil which together represents a granular soil does reduce the amplitude and increase the frequency of vibration. However, the presence of a viscous liquid in the transparent soil might be offsetting the increase in frequency of vibration. Hence, the expectation would be that the frequency of vibration in soil would be more than 11.38 kHz. This requires further study on the effects of viscosity on BE vibration and will be addressed in a future study.

5.3 Variation of BE response with increasing vertical stress

BE response is measured for five different vertical stresses to observe the change in BE vibrations with vertical stress. Vertical stress increase was induced by adding multiple loads in increments of 2.5 kg. Stress increase at each load level is calculated using the Boussinesq (1885) point load solution. Loads and the corresponding stresses are shown in Table 1. The diameter of the top platen is 7 cm (resonant column sample diameter) while the depth at which the stress levels are obtained is 2 mm (half of BE width).

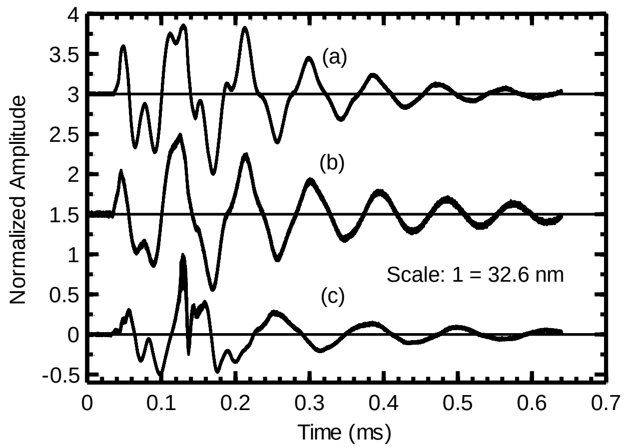

Figure 7 - BE response for different media; (a) in air, (b) in air with plexiglass in between, and (c) in oil

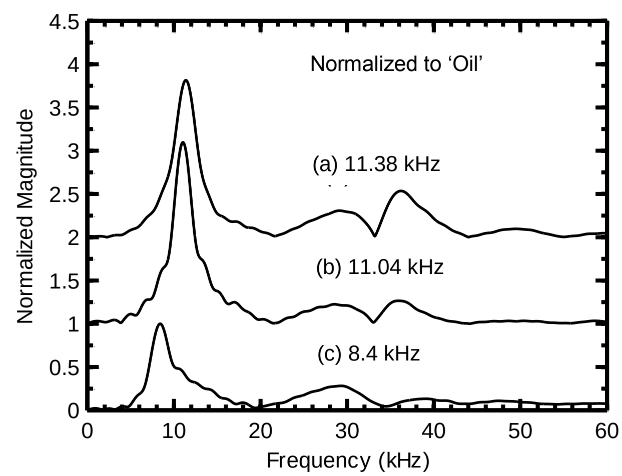

Figure 8 - FFT magnitude of BE response for different media; (a) in air, (b) in air with plexiglass in between, and (c) in oil

Table 1 - Stress increase corresponding to each dead load

Load increase (kg)	Stress increase, Δσ (kPa)
0[(1)]	2.1
2.5	8.5
5	14.9
7.5	21.2
10	27.6

[1]stress is due to the weight of platen and tube

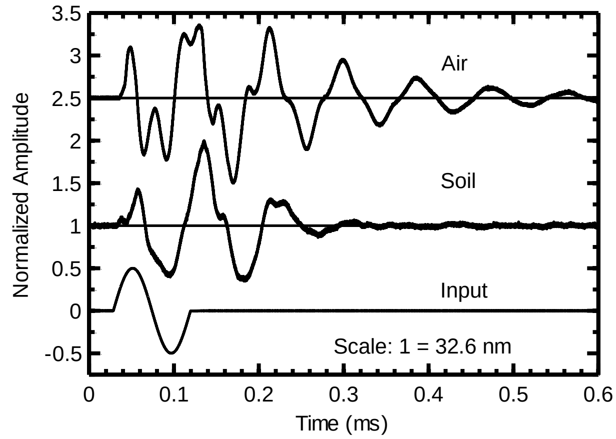

Figure 9 - BE responses in air and in transparent soil

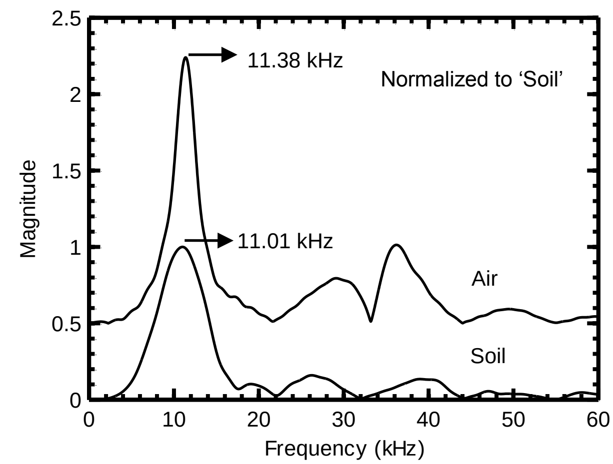

Figure 10 - FFT magnitude of BE responses in air and in soil

Figures 11 and 12 show, respectively, the time signals and FFT magnitudes of BE response corresponding to the stresses of 0 and 10 kg in Table 1. Time signals show that the waveform of vibration does not change significantly with increase in stress level. Variation of frequency at peak magnitude with confining pressure is shown in Figure 13. A measurement is also obtained for BE vibration after the applied loads are removed. The results show that the resonance frequency of BE vibration immersed in soil increases with increase in vertical stress. However, this increase is not linear. The natural frequency increases with increase in vertical stress because the BE is vibrating in a more compact medium as the vertical stress increases. This should be corroborated by a decrease in amplitude of BE vibration. However, Figure 14 shows otherwise. The variation of peak amplitude at resonance frequency with vertical stress does not show the expected trend. The amplitude increases for loads from 0 to 5 kg and decreases from 5 to 10 kg. A possible reason for this could be that the amplitude at resonance frequency of BE vibration in soil has an optimum value of stress at which the highest amplitude is achieved.

However, this is only a prediction. Further study is required to understand this unusual variation of peak magnitudes with vertical stress increase.

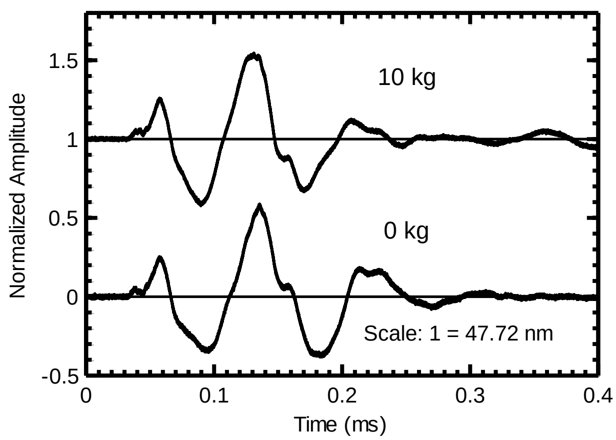

Figure 11 – BE responses under two stresses

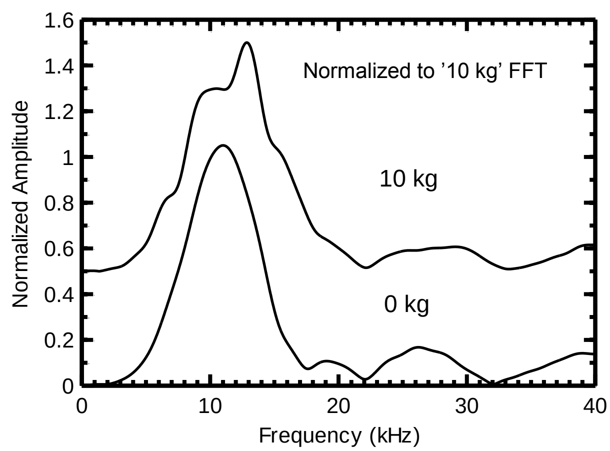

Figure 12 – FFT magnitude of BE responses under two stresses

6 CONCLUSIONS

BE are commonly used to estimate V_s of soils. However, the BE test remains without a standard procedure mainly because of the difficulty in controlling the actual behavior of BE in a soil specimen. This study presents results of novel experimental program being conducted to evaluate the actual behavior of BE in a soil specimen. A transparent soil along with a state-of-the-art laser vibrometer was used for measuring BE vibrations in the transparent soil.

The results showed that the maximum response in the transmitter BE is obtained when a sine pulse excitation is used at the natural frequency of the BE. The square pulse excitation induced a sharper response of the BE transmitter; however, it induced significant deformations at high frequencies.

The 11 kHz sine pulse excitation was used because it did not induce significant energy in high frequency of BE vibrations. The square pulse excitations induced high frequencies in BE response. These high frequencies were not associated with higher resonance frequency mode shapes of BE. Measurements at different points on BE showed that the vibrations on different points are out of phase by about 0.95 radians. Measurements in transparent soil showed that the increase in natural frequency of BE vibration in soil was being offset by the presence of the viscous mineral oil. Variation of natural frequency of BE with stresses showed an increase of natural frequency with stress increase albeit not linear. However, magnitude at resonance frequency showed an unusual variation with stress increase. This result and the effect of viscous liquids on BE vibrations will be further investigated in subsequent studies of this experimental program.

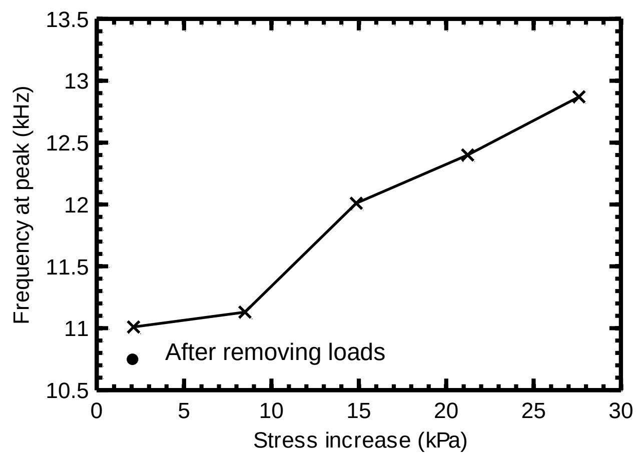

Figure 13 - Variation of frequency at peak of BE vibration with stress increase

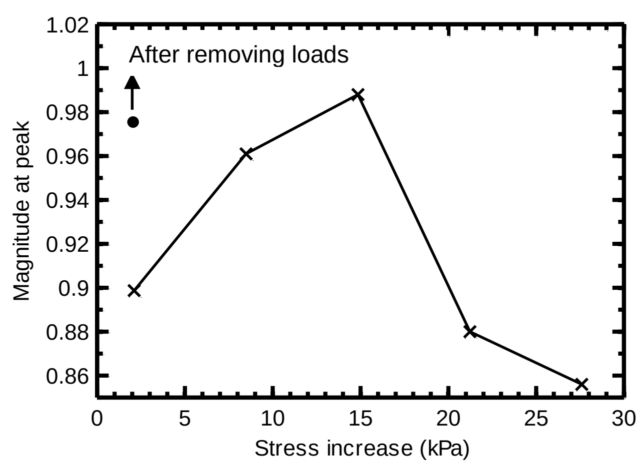

Figure 14 - Variation of amplitude at natural frequency of BE vibration with stress increase

7 ACKNOWLEDGEMENTS

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8 REFERENCES

Arroyo, M., Wood, D. M., Greening, P. D., Medina, L., & Rio, J. (2006). Effects of sample size on bender-based axial G0 measurements. Géotechnique, 56(1), 39-52.

Arulnathan, R., Boulanger, R. W., & Riemer, M. F. (1998). Analysis of bender element tests. ASTM Geotechnical Testing Journal, 21(2), 120-131.

Brignoli, E., Gotti, M., & Stokoe, K. H. (1996). Measurement of shear waves in laboratory specimens by means of piezoelectric transducers. ASTM Geotechnical Testing Journal, 19(4), 384-397.

Camacho-Tauta, J., Santos, J., Ferreira, C., Viana, D., & Fonseca, A. (2008). Moving windows algorithm to reduce uncertainties in bender element testing. XI Portuguese National Congress of Geotechnics, Coimbra, , 1 149-156.

Cascante, G., Vanderkooy, J., & Chung, W. (2005). A new mathematical model for resonant-column measurements including eddy-current effects. Canadian Geotechnical Journal, 42(1), 121-135.

Clough, R. W., & Penzien, J. (2003). Dynamics of Structures. Berkeley, CA, USA: Computers & Structures.

Dyvik, R., & Madshus, C. (1985). Laboratory measurements of G^ using bender elements. Proc.of ASCE Annual Convention, Advances in the Art of Testing Soils Under Cyclic Conditions, , 186-196.

Ezzein, F. M., & Bathurst, R. J. (2011). A transparent sand for geotechnical laboratory modeling. ASTM Geotechnical Testing Journal, 34(6), 590-601.

Greening, P. D., & Nash, D. F. (2004). Frequency Domain Determination of G~ 0 Using Bender Elements. Geotechnical Testing Journal, 27(3), 288-294.

Ismail, M., & Hourani, Y. (2003). An innovative facility to measure shear-wave velocity in centrifuge and 1-g models. Proceedings of Deformation Characteristics of Geomaterials, Lyon, , 21-29.

Jovicic, V., Coop, M., & Simic, M. (1996). Objective criteria for determining Gmax from bender element tests. Geotechnique, 46(2), 357-362.

Lee, J., & Santamarina, J. C. (2005). Bender elements: performance and signal interpretation. Journal of Geotechnical and Geoenvironmental Engineering, 131(9), 1063-1070.

Pallara, O. V., Mattone, M. C., & Lo Presti, D. C. (2008). Bender elements: bad source-good receiver. Deformational Characteristics of Geomaterials, , 697-702.

Rio, J. F. M. E. (2006). Advances in laboratory geophysics using bender elements (Ph.D.). Available from ProQuest Dissertations & Theses: UK & Ireland. (301675421).

Santamarina, J. C., & Fam, M. A. (1997). Discussion: Interpretation of bender element tests. Geotechnique, 47(4), 873-877.

Shirley, D. J., & Hampton, L. D. (1978). Shear-wave measurements in laboratory sediments. The Journal of the Acoustical Society of America, 63(2), 607-613.

Tallavo, F., Cascante, G., & Pandey, M. D. (2009). New methodology for source characterization in pulse velocity testing. ASTM Geotechnical Testing Journal, 32(6), 537-552.

Viggiani, G., & Atkinson, J. (1995). Interpretation of bender element tests. International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts, , 32(8)

Weast, R., Astle, M., & Beyer, W. (1982). CRC Handbook of Chemistry and Physics CRC, Boca Raton, FL (1981), pp. E-103,